Decision Making with Assumption-based Argumentation

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Abstract. In this paper, we present two different formal frameworks for representing decision making. In both frameworks, decisions have multiple attributes and meet different goals. In the second framework, decisions take into account preferences over goals. We also study a family of decision functions representing making decisions with different criteria, including decisions meeting all goals, most goals, goals no other decisions meet, and most preferred achievable goals. For each decision function, we define an argumentation-based computational mechanism for computing and explaining the selected decisions. We make connections between decision making and argumentation semantics, i.e., selected decisions in a decision making framework are admissible arguments in the corresponding argumentation framework. The main advantage of our approach is that it not only selects decisions but gives an argumentation-based justification of selected decisions.

1 Introduction

Argumentation based decision making has attracted considerable research interest in recent years [1, 7, 6, 9]. In this paper, we give a formal treatment of two forms of decision making with argumentation. We view decision making as concerned with three related processes: (I) agents represent information that is relevant to the decision making; (II) agents choose a decision criteria to represent "good" decisions; and (III) agents compute and explain the desired decision based on the selected criteria. We realise these three components formally.

We give formal definitions for *decision frameworks*, used to model the agents' knowledge bases to support I. We allow a decision framework to have multiple *decisions* and a set of *goals*, such that each decision can have a number of different *attributes* and each goal can be satisfied by some attributes. With decision frameworks defined, we model different decision criteria with *decision functions.* Given a decision framework, decision functions return a set of *selected* decisions, representing decisions that meet the decision criteria underpinning the decision function. To compute and explain the desired decisions, we map decision frameworks and decision functions into assumption-based argumentation (ABA) frameworks [3]. We prove that selected decisions w.r.t. a given decision function are claims of admissible arguments in the corresponding ABA framework. The main advantage of our approach is that while finding the "good" decisions, it gives an argumentation-based justification of selected decisions.

This paper is organised as follows. We briefly introduce ABA in Section 2. We define decision frameworks and decision functions in Section 3. We present ABA representation of decision frameworks and functions in Section 4. We introduce decision making with preference over goals in Section 5. We review a few related work in Section 6. We conclude in Section 7.

2 Background

An ABA framework [3, 5] is a tuple $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ where

- $\langle \mathcal{L}, \mathcal{R} \rangle$ is a deductive system, with $\mathcal{L}$ the *language* and $\mathcal{R}$ a set of *rules* of the form $s_0 \leftarrow s_1, \ldots, s_m (m \geq 0)$;
- $\mathcal{A} \subseteq \mathcal{L}$ is a (non-empty) set, referred to as *assumptions*;
- $\mathcal{C}$ is a total mapping from $\mathcal{A}$ into $2^{\mathcal{L}}$, where $\mathcal{C}(\alpha)$ is the *contrary* of $\alpha \in \mathcal{A}$.

When presenting an ABA framework, we omit presenting $\mathcal{L}$ explicitly as we assume $\mathcal{L}$ contains all sentences appearing in $\mathcal{R}$, $\mathcal{A}$ and $\mathcal{C}$. Given a rule $s_0 \leftarrow s_1, \ldots, s_m$, we use the following notation: $Head(s_0 \leftarrow s_1, \ldots, s_m) = s_0$ and $Body(s_0 \leftarrow s_1, \ldots, s_m) = \{s_1, \ldots, s_m\}$. As in [3], we enforce that ABA frameworks are *flat*, namely assumptions do not occur in the head of rules.

In ABA, *arguments* are deductions of claims using rules and supported by assumptions, and *attacks* are directed at assumptions. Informally, following [3]:

- *an argument for (the claim)* $c \in \mathcal{L}$ *supported by* $S \subseteq \mathcal{A}$ ($S \vdash c$ in short) is a (finite) tree with nodes labelled by sentences in $\mathcal{L}$ or by the symbol τ[1], such that the root is labelled by c, leaves are either τ or assumptions in S, and non-leaves s have as many children as elements in the body of a rule with head s, in a one-to-one correspondence with the elements of this body.
- *an argument* $S_1 \vdash c_1$ *attacks an argument* $S_2 \vdash c_2$ iff $c_1 = \mathcal{C}(\alpha)$ for $\alpha \in S_2$.

Attacks between arguments correspond in ABA to attacks between sets of assumptions, where *a set of assumptions* A *attacks a set of assumptions* B iff an argument supported by $A' \subseteq A$ attacks an argument supported by $B' \subseteq B$.

When there is no ambiguity, we also say a sentence b attacks a sentence a when a is an assumption and b is a claim of an argument $\texttt{Arg}'$ such that a is in the support of some argument $\texttt{Arg}$ and $\texttt{Arg}'$ attacks $\texttt{Arg}$.

With argument and attack defined, standard argumentation semantics can be applied in ABA [3]. We focus on the admissibility semantics: *a set of assumptions is admissible* (in $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$) iff it does not attack itself and it attacks all $A \subseteq \mathcal{A}$ that attack it; *an argument* $S \vdash c$ *belongs to an admissible extension supported by* $\Delta \subseteq \mathcal{A}$ (in $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$) iff $S \subseteq \Delta$ and Δ is admissible. When there is no ambiguity, we also say an argument $\texttt{Arg}$ is admissible if $\texttt{Arg}$ belongs to an admissible extension supported by some Δ.

[1] As in [3], $\tau \notin \mathcal{L}$ stands for "true" and is used to represent the empty body of rules.

3 Decision Frameworks and Decision Functions

In this paper, we consider the following form of decision: there are a set of possible decisions $\mathtt{D}$, a set of attributes $\mathtt{A}$, and a set of goals $\mathtt{G}$, such that a decision $d \in \mathtt{D}$ may *have* some attributes $A \subseteq \mathtt{A}$, and each goal $g \in \mathtt{G}$ is *satisfied* by some attributes $A' \subseteq \mathtt{A}$. Then decisions can be selected based on a certain *decision function*. The relations between decisions and attributes and between goals and attributes jointly form a *decision framework*, which can be represented as two tables, as follows:

Definition 1. *A* decision framework *is a tuple* $\langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, consisting of:*

- *a set of decisions* $\mathtt{D} = \{d_1, \ldots, d_n\}, n > 0$*,*
- *a set of attributes* $\mathtt{A} = \{a_1, \ldots, a_m\}, m > 0$*,*
- *a set of goals* $\mathtt{G} = \{g_1, \ldots, g_l\}, l > 0$*, and*
- *two tables:* $\mathtt{T_{DA}}$*, of size* $(n \times m)$*, and* $\mathtt{T_{GA}}$*, of size* $(l \times m)$*, such that*
  - *for every* $\mathtt{T_{DA}}[\mathtt{i}, \mathtt{j}]$[2]$, 1 \leq i \leq n, 1 \leq j \leq m$*,* $\mathtt{T_{DA}}[\mathtt{i}, \mathtt{j}]$ *is either* 1*, representing that decision* d_i has *attributes* a_j*, or* 0*, otherwise;*
  - *for every* $\mathtt{T_{GA}}[\mathtt{i}, \mathtt{j}], 1 \leq i \leq l, 1 \leq j \leq m$*,* $\mathtt{T_{GA}}[\mathtt{i}, \mathtt{j}]$ *is either* 1*, representing that goal* g_i *is* satisfied *by attribute* a_j*, or* 0*, otherwise.*

We assume that the column order in both $\mathtt{T_{DA}}$ *and* $\mathtt{T_{GA}}$ *is the same, and the* indices *of decisions, goals, and attributes in* $\mathtt{T_{DA}}$ *and* $\mathtt{T_{GA}}$ *are the row numbers of the decision and goals and the column number of attributes in* $\mathtt{T_{DA}}$ *and* $\mathtt{T_{GA}}$*, respectively. We use* $\mathcal{DEC}$ *and* $\mathcal{DF}$ *to denote the set of all possible decisions and the set of possible decision frameworks, respectively.*

The notion of Decision frameworks is illustrated as follows, adopted from [8].

Example 1. An agent needs to decide an accommodation in London. The two tables, $\mathtt{T_{DA}}$ and $\mathtt{T_{GA}}$, are given in Table 1.

	£50	£70	£200	inSK	inPic
jh	0	1	0	1	0
ic	1	0	0	1	0
ritz	0	0	1	0	1

	£50	£70	£200	inSK	inPic
cheap	1	0	0	0	0
near	0	0	0	1	0

Table 1. $\mathtt{T_{DA}}$(left) and $\mathtt{T_{GA}}$(right).

Decision ($\mathtt{D}$) are: hotel (jh), Imperial College Halls (ic), Ritz (ritz). Attributes ($\mathtt{A}$) are: £50, £70, £200, in South Kensington (inSK), and in Piccadilly (inPic). Goals ($\mathtt{G}$) are: cheap and near. The indices are: 1-jh; 2-ic; 3-ritz; 1-cheap; 2-near; 1-£50; 2-£70; 3-£200; 4-inSK; 5-inPic. In this example, jh is £70 and is in South Kensington; ic is £50 and is in South Kensington; ritz is £200 and is in Piccadilly; £50 is cheap and accommodations in South Kensington are near.

We define a decision *meeting* a goal as the follows:

[2] We use $\mathtt{T_X}[\mathtt{i}, \mathtt{j}]$ to represent the cell in row i and column j in $\mathtt{T_X} \in \{\mathtt{T_{DA}}, \mathtt{T_{GA}}\}$.

Definition 2. *Given* $\langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, a decision* $d \in \mathtt{D}$ *with row index* i *in* $\mathtt{T_{DA}}$ meets *a goal* $g \in \mathtt{G}$ *with row index* j *in* $\mathtt{T_{GA}}$ *iff there exists an attribute* $a \in \mathtt{A}$ *with column index* k *in both* $\mathtt{T_{DA}}$ *and* $\mathtt{T_{GA}}$*, such that* $\mathtt{T_{DA}}[\mathtt{i}, \mathtt{k}] = 1$ *and* $\mathtt{T_{GA}}[\mathtt{j}, \mathtt{k}] = 1$*.*

We use $\gamma(d) = S$*, where* $d \in \mathtt{D}, S \subseteq \mathtt{G}$*, to denote the set of goals met by* d*.*

In Example 1, *jh* meets *near*; *ic* meets *cheap* and *near*; *ritz* meets no goal.

Decision frameworks provide information for decision making. Given a decision framework, a *decision function* returns the set of "good" decisions. Formally,

Definition 3. *A* decision function *is a mapping* $\psi : \mathcal{DF} \mapsto 2^{\mathcal{DEC}}$*, such that, given* $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*,* $\psi(df) \subseteq \mathtt{D}$*. For any* $d, d' \in \mathtt{D}$*, if* $\gamma(d) = \gamma(d')$ *and* $d \in \psi(df)$*, then* $d' \in \psi(df)$*. We say that* $\psi(df)$ *are* selected *in* $\mathcal{DF}$ *w.r.t.* ψ*.*

We use Ψ *to denote the set of all decision functions.*

Definition 3 defines that if two decisions meet the same set of goals and a decision function selects one of the decisions, then the decision function must select the other decision as well.

We subsequently define three decision functions, each characterising a notion of "good decision". They all fulfil the requirement in Definition 3 but also characterise additional requirements. We start with the notion of *strongly dominant decision functions* that select the decisions meeting all goals. Formally,

Definition 4. *A* strongly dominant decision function $\psi \in \Psi$ *is such that given* $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, for all decisions* $d \in \psi(df)$*,* $\gamma(d) = \mathtt{G}$*. We say that any such* d *is a* strongly dominant decision*.*

We refer to a generic strongly dominant decision function as ψ_s*.*

In Example 1, *ic* is a strongly dominant decision as it meets both *cheap* and *near*. There is no other strongly dominant decision.

Strongly dominant decisions can be relaxed to *dominant decisions* which meet all goals that are ever met by any decision in the decision framework. Formally,

Definition 5. *A* dominant decision function $\psi \in \Psi$ *is such that given* $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, for any* $d \in \psi(df)$*, let* $S = \gamma(d)$*, then there is no* $g' \in \mathtt{G} \setminus S$ *and* $g' \in \gamma(d')$*, where* $d' \in \mathtt{D} \setminus \{d\}$*. We say such* d *is a* dominant decision*.*

We refer to a generic dominant decision function as ψ_d*.*

In Example 1, *ic* is a dominant decision. There is no other dominant decision. To illustrate the case when there is no strongly dominant decision, but only dominant decisions, we introduce the following example.

Example 2. We again consider an agent deciding accommodation in London. $\mathtt{T_{DA}}$ and $\mathtt{T_{GA}}$ are given in Table 2. Unlike Example 1, there is no decision *ic* that meets both goals, *cheap* and *near*. Nevertheless, *jh* is a better decision than *ritz* as it meets *near* whereas *ritz* meets no goal. Hence, in this example, there is no strongly dominant decision, but there is a dominant decision, *jh*.

By Definition 5, all dominant decisions meet the same set of goals, formally:

	£50	£70	£200	inSK	inPic
jh	0	1	0	1	0
ritz	0	0	1	0	1

	£50	£70	£200	inSK	inPic
cheap	1	0	0	0	0
near	0	0	0	1	0

Table 2. $\mathtt{T_{DA}}$(left) and $\mathtt{T_{GA}}$(right).

Proposition 1. *Given df* $\in \mathcal{DF}$*, for any* $d, d' \in \psi_d(df)$*,* $\gamma(d) = \gamma(d')$*.*

Moreover, if all decisions meet the same set of goals, then they are dominant.

Lemma 1. *Given df* $= \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, if for all* $d, d' \in \mathtt{D}$*,* $\gamma(d) = \gamma(d')$*, then* $\psi_d(df) = \mathtt{D}$*.*

Trivially, strongly dominant decisions are also dominant.

Proposition 2. *Given df* $\in \mathcal{DF}$*,* $\psi_s(df) \subseteq \psi_d(df)$*.*

Dominant decisions can be weakened to *weakly dominant*. Goals met by a weakly dominant decision is not a subset of goals met by some other decision.

Definition 6. *A* weakly dominant decision function $\psi \in \Psi$ *is such that given df* $= \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, for all* $d \in \psi(df)$*, there is no* $d' \in \mathtt{D} \setminus \{d\}$ *and* $\gamma(d) \subset \gamma(d')$*.*
We refer to a generic weakly dominant decision function as ψ_w*.*

In Example 1, *ic* is weakly dominant; there is no other weakly dominant decision. In Example 2, *jh* is weakly dominant; there is no other weakly dominant decision. To illustrate the case when there is no dominant decision but only weakly dominant decisions, we introduce the next example.

Example 3. (Continue Example 1). The new $\mathtt{T_{DA}}$ and $\mathtt{T_{GA}}$ shown in Table 3.

	£50	£70	£200	inSK	inPic
jh	0	1	0	1	0
ic	1	0	0	0	0
ritz	0	0	1	0	1

	£50	£70	£200	inSK	inPic
cheap	1	0	0	0	0
near	0	0	0	1	0

Table 3. $\mathtt{T_{DA}}$(left) and $\mathtt{T_{GA}}$(right).

Unlike Example 1, *ic* no longer meets *near*. Hence *ic* is not strongly dominant. However, *ic* meets *cheap*, which is not met by *jh*, so *jh* is not dominant as in Example 2. Since *ic* and *jh* both meet goals that are not met by the other, the are both weakly dominant. *ritz* meets no goal and is not weakly dominant.

Trivially, a dominant decision is also weakly dominant.

Proposition 3. *Given df* $\in \mathcal{DF}$*,* $\psi_d(df) \subseteq \psi_w(df)$*.*

If a set of decisions S is strongly dominant, then S is also dominant and weakly dominant; there is no other dominant or weakly dominant decision.

Proposition 4. *Given df* $\in \mathcal{DF}$*, let* $S_s = \psi_s(df)$*,* $S_d = \psi_d(df)$*, and* $S_w = \psi_w(df)$*, if* $S_s \neq \{\}$*, then* $S_s = S_d = S_w$*.*

Proof. First we prove $S_s = S_d$. By Proposition 2, $S_s \subseteq S_d$. We show that there is no d such that $d \in S_d$, $d \notin S_s$. Assuming otherwise, (1) since $d \notin S_s$, $\gamma(d) \neq \mathtt{G}$, hence there is some $g \in \mathtt{G}$ and $g \notin \gamma(d)$; (2) since $S_s \neq \{\}$, there is $d' \in S_s$ such that $\gamma(d') = \mathtt{G}$, therefore $g \in \gamma(d')$. By (1) and (2), $d \notin S_d$. Contradiction.

Then we prove $S_s = S_w$. Similarly, we assume $S_s \subset S_w$. Since $S_s \subset S_w$, there exists $d \in S_w$, $d \notin S_s$. Since $S_s \neq \{\}$, there exists $d' \in S_s$ and $\gamma(d') = \mathtt{G}$. Since $d \notin S_s$, $\gamma(d) \subset \mathtt{G}$. Hence $\gamma(d) \subset \gamma(d')$. By Definition 6, $d \notin S_w$. Contradiction.

Similarly, if there exists a non-empty set of dominant decisions S, then there is no weakly dominant decisions other than S. Formally:

Proposition 5. *Given $df \in \mathcal{DF}$, let $S_d = \psi_d(df)$ and $S_w = \psi_w(df)$. If $S_d \neq \{\}$, then $S_d = S_w$.*

Proof. By Proposition 3, we know $S_d \subseteq S_w$. We show $S_w \subseteq S_d$. Assume otherwise, i.e., there exists $d \in S_w$ and $d \notin S_d$. Since $S_d \neq \{\}$, there exists $d' \in S_d$, such that $\gamma(d') \supseteq \gamma(d'')$, for all $d'' \in \mathtt{D}$. Hence $\gamma(d) \subseteq \gamma(d')$. Since $d \notin S_d$, $\gamma(d) \neq \gamma(d')$. Therefore, $\gamma(d) \subset \gamma(d')$. By Definition 6, $d \notin S_w$. Contradiction.

As illustrated by Example 3, given a decision framework *df*, if there is no dominant decision in *df*, but only weakly dominant decisions S, then S contains at least two decisions such that each meets a different set of goals.

Theorem 1. *Given $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$, let $S_d = \psi_d(df)$ and $S_w = \psi_w(df)$. If $S_d = \{\}$ and $S_w \neq \{\}$, then there exists $d, d' \in S_w, d \neq d'$ and $\gamma(d) \neq \gamma(d')$.*

Proof. Since $S_d = \{\}$, by Lemma 1, $|\mathtt{D}| > 1$. Assume that for all $d, d' \in S_w, d \neq d', \gamma(d) = \gamma(d')$. Then there are two cases, both of them leading to contradictions.

1. First case, if there is no $d'' \in \mathtt{D} \setminus S_w$, then $S_w = \mathtt{D}$. Since $\gamma(d) = \gamma(d')$ for all d, d', by Lemma 1, for all $d \in S_w, d \in \psi_d(df)$, but $S_d = \{\}$. Contradiction.
2. Second case, if there exists some $d'' \in \mathtt{D} \setminus S_w$. Then there are five possibilities between $\gamma(d)$ and $\gamma(d'')$, and they all give contradictions, as follows:
   (a) $\gamma(d) \supset \gamma(d'')$. Not possible, as if so there would exists $d^* \in \mathtt{D}$ such that d^* is dominant (d could be a candidate for such d^*).
   (b) $\gamma(d) \subset \gamma(d'')$. Not possible, as if so there would exists $d^* \in \mathtt{D} \setminus S_w$ such that d^* is dominant (d'' could be a candidate for such d^*).
   (c) $\gamma(d) = \gamma(d'')$. Not possible, as if so d'' would be in S_w.
   (d) None of (a)(b)(c) but $\gamma(d) \cap \gamma(d'') \neq \{\}$. Not possible, as if so there would exist $g \in \gamma(d''), g \notin \gamma(d)$, hence there would exist $d^* \in \mathtt{D} \setminus S_w$ and d^* is weakly dominant (d'' could be a candidate for such d^*), but $\psi_w(df) = S_w$ and $d^* \notin S_w$.
   (e) None of (a)(b)(c), but $\gamma(d) \cap \gamma(d'') = \{\}$. Same as case 2(d).

Both case 1 and 2 give contradictions, this theorem holds.

Theorem 1 gives an important result. Comparing with Definition 4 ($\gamma(d) = \mathtt{G}$ for all $d \in \psi_s$) and Proposition 1 ($\gamma(d) = \gamma(d')$ for all $d, d' \in \psi_d$), showing that (strongly) dominant decisions meet the same goals, Theorem 1 shows that weakly dominant decisions meet different goals. Hence, selecting different decisions from a (strongly) dominant set makes no difference w.r.t. the decision maker, whereas selecting different decisions from a weakly dominant decision set would.

4 Computing and Explaining Decisions with ABA

As seen in [8], ABA can be used to compute and explain decisions. Given a decision framework and a decision function, we can construct an ABA framework, AF, in a way such that admissible arguments in AF are selected decisions.

We introduce *strongly dominant ABA frameworks* to compute strongly dominant decisions in a decision framework. Formally,

Definition 7. *Given a decision framework* $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, in which* $|\mathtt{D}| = n$*,* $|\mathtt{A}| = m$ *and* $|\mathtt{G}| = l$*, the* strongly dominant ABA framework corresponding to $\langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$ *is* $df_S = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$*, where*

- $\mathcal{R}$ *is such that: for all* $k = 1, .., n; j = 1, .., m$ *and* $i = 1, .., l$*:*
  - *if* $\mathtt{T_{DA}}[\mathtt{k}, \mathtt{i}] = 1$ *then* $d_k a_i \leftarrow$*;*
  - *if* $\mathtt{T_{GA}}[\mathtt{j}, \mathtt{i}] = 1$ *then* $g_j a_i \leftarrow$*;*
  - $d_k g_j \leftarrow d_k a_i, g_j a_i$*;*
- $\mathcal{A}$ *is such that:* d_k*, for* $k = 1, .., n$*;* $N d_k g_j$*, for* $k = 1, .., n$ *and* $j = 1, .., m$*;*
- $\mathcal{C}$ *is such that:* $\mathcal{C}(d_k) = \{N d_k g_1, \ldots, N d_k g_n\}$*, for* $k = 1, .., n$*;*
  $\mathcal{C}(N d_k g_j) = \{d_k g_j\}$*, for* $k = 1, .., n$ *and* $j = 1, .., m$*.*

The intuition behind Definition 7 is as follows: given a decision d_k, we let d_k be an assumption. We check if d_k meets all goals by defining the contrary of d_k to be $\{N d_k g_1, \ldots, N d_k g_m\}$ (standing for d_k does not meet g_1, …, d_k does not meet g_m). Each of these "negative" assumption is then attacked by a "proof" that d_k meets g_j, i.e., a "proof" for $d_k g_j$. From Definition 2, we know that d_k meets g_j iff there is an attribute a_i such that d_k has a_i and g_j is satisfied by a_i. Hence, we check in both $\mathtt{T_{DA}}$ and $\mathtt{T_{GA}}$ to see if such a_i exists.

We illustrate the notion of strongly dominant ABA framework corresponding to a decision framework in the following example.

Example 4. (Continue Example 1.) Given the decision framework *df* in Example 1, $df_S = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ has

$$\mathcal{R} \text{ (rules): } \begin{array}{llll} jh70 \leftarrow & jhSK \leftarrow & ic50 \leftarrow & icSK \leftarrow \\ ritz200 \leftarrow & ritzPic \leftarrow & cheap50 \leftarrow & nearSK \leftarrow \end{array}$$

$$\begin{array}{ll}
jhCheap \leftarrow jh50, cheap50 & jhNear \leftarrow jh50, near50 \\
jhCheap \leftarrow jh70, cheap70 & jhNear \leftarrow jh70, near70 \\
jhCheap \leftarrow jh200, cheap200 & jhNear \leftarrow jh200, near200 \\
jhCheap \leftarrow jhSK, cheapSK & jhNear \leftarrow jhSK, nearSK \\
jhCheap \leftarrow jhPic, cheapPic & jhNear \leftarrow jhPic, nearPic \\
icCheap \leftarrow ic50, cheap50 & icNear \leftarrow ic50, near50 \\
icCheap \leftarrow ic70, cheap70 & icNear \leftarrow ic70, near70 \\
icCheap \leftarrow ic200, cheap200 & icNear \leftarrow ic200, near200 \\
icCheap \leftarrow icSK, cheapSK & icNear \leftarrow icSK, nearSK \\
icCheap \leftarrow icPic, cheapPic & icNear \leftarrow icPic, nearPic \\
ritzCheap \leftarrow ritz50, cheap50 & ritzNear \leftarrow ritz50, near50 \\
ritzCheap \leftarrow ritz70, cheap70 & ritzNear \leftarrow ritz70, near70 \\
ritzCheap \leftarrow ritz200, cheap200 & ritzNear \leftarrow ritz200, near200 \\
ritzCheap \leftarrow ritzSK, cheapSK & ritzNear \leftarrow ritzSK, nearSK \\
ritzCheap \leftarrow ritzPic, cheapPic & ritzNear \leftarrow ritzPic, nearPic
\end{array}$$

	jh	ic	$ritz$
$\mathcal{A}$ (assumptions):	$NjhCheap$	$NicCheap$	$NritzCheap$
	$NjhNear$	$NicNear$	$NritzNear$

$\mathcal{C}$ (contraries):

$\mathcal{C}(jh) = \{NjhCheap, NjhNear\}$
$\mathcal{C}(ic) = \{NicCheap, NicNear\}$
$\mathcal{C}(ritz) = \{NritzCheap, NritzNear\}$

$\mathcal{C}(NjhCheap) = \{jhCheap\}$	$\mathcal{C}(NjhNear) = \{jhNear\}$
$\mathcal{C}(NicCheap) = \{icCheap\}$	$\mathcal{C}(NicNear) = \{icNear\}$
$\mathcal{C}(NritzCheap) = \{ritzCheap\}$	$\mathcal{C}(NritzNear) = \{ritzNear\}$

Formally, we show the correspondence between strongly dominant decisions and the ABA counterpart as follows.

Theorem 2. *Given $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$, let df_S be the strongly dominant ABA framework corresponding to df. Then for all decisions $d \in \mathtt{D}$, $d \in \psi_s(df)$ iff $\{d\} \vdash d$ is admissible in df_S.*

Proof. Let d be d_k (k is the index of d in $\mathtt{T_{DA}}$). We first prove if d_k is strongly dominant, then $\{d_k\} \vdash d_k$ is admissible. Since d_k is strongly dominant, $\gamma(d_k) = \mathtt{G}$. Hence, for every $g \in \mathtt{G}$, d_k meets g. Therefore, for every $g \in \mathtt{G}$, there exists some $a \in \mathtt{A}$, such that d_k has a and g is satisfied by a. Let the indices of g and a be j and i, in both $\mathtt{T_{DA}}$ and $\mathtt{T_{GA}}$, respectively, then $\mathtt{T_{DA}}[\mathtt{k}, \mathtt{i}] = \mathtt{T_{GA}}[\mathtt{j}, \mathtt{i}] = 1$. Hence, $d_k a_i \leftarrow$ and $g_j a_i \leftarrow$ are in $\mathcal{R}$ for all j. Therefore $\{\} \vdash d_k g_j$ exists for all j and are not attacked. Hence, $\{Nd_k g_j\} \vdash Nd_k g_j$ is attacked for all j; and since $\{d_k\}$ is conflict-free, $\{d_k\} \vdash d_k$ is admissible.

We then show if $\{d_k\} \vdash d_k$ is admissible then d_k is strongly dominant. Let $\{Nd_k g_j\} \vdash Nd_k g_j$ be attackers of $\{d_k\} \vdash d_k$. Since $\{d_k\} \vdash d_k$ is admissible, it withstands all of its attacks. Hence, $\{Nd_k g_j\} \vdash Nd_k g_j$ must be attacked for all j. Since $\mathcal{C}(Nd_k g_j) = \{d_k g_j\}$, $\{\} \vdash d_k g_j$ must exist for all j. Because the only rule with head $d_k g_j$ is $d_k g_j \leftarrow d_k a_i, g_j a_i$, for each j there exists some i such that $d_k a_i \leftarrow$ and $g_j a_i \leftarrow$. Then, for each j there must exist some i such that $\mathtt{T_{DA}}[\mathtt{k}, \mathtt{i}] = \mathtt{T_{GA}}[\mathtt{j}, \mathtt{i}] = 1$ for all j. Therefore d meets all goals g in $\mathtt{G}$ and d is strongly dominant.

The relation between strongly dominant decisions and admissible arguments in strongly dominant ABA framework is shown in the following example.

Example 5. (Continue Example 4.) Given the decision framework *df* in Example 1, and the strongly dominant ABA framework $df_S = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ in Example 4, we see that $\{ic\} \vdash ic$ is admissible, as its attackers $\{NicCheap\} \vdash NicCheap$ and $\{NicNear\} \vdash NicNear$ are both attacked by $\{\} \vdash icCheap$ and $\{\} \vdash icNear$, respectively. The argument $\{\} \vdash icNear$ is admissible as $icNear \leftarrow icSK, nearSK$; $icSK \leftarrow$ and $nearSK \leftarrow$ are in $\mathcal{R}$. Similarly, $\{\} \vdash icCheap$ is admissible as $icCheap \leftarrow ic50, cheap50$; $ic50 \leftarrow$ and $cheap50 \leftarrow$ are in $\mathcal{R}$ and there is no argument attacks $\{\} \vdash icNear$ or $\{\} \vdash icCheap$. The graphical illustration is shown in Figure 1.

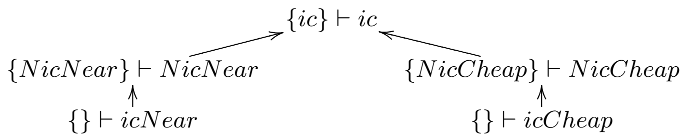

Fig. 1. Graphical illustration of Example 5. Here, $\{ic\} \vdash ic$ is admissible as it is an argument and its attackers $\{NicNear\} \vdash NicNear$ and $\{NicCheap\} \vdash NicCheap$ are both counterattacked.

Given a decision framework, dominant decisions can also be computed with ABA in its corresponding *dominant ABA framework*. Formally,

Definition 8. *Given* $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*,* $|\mathtt{D}| = n$*, and* $|\mathtt{A}| = m$*, let the corresponding strongly dominant ABA framework be* $df_S = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$*, then the* dominant ABA framework corresponding to df *is* $df_D = \langle \mathcal{L}, \mathcal{R}_D, \mathcal{A}_D, \mathcal{C}_D \rangle$*, where:*

- $\mathcal{R}_D = \mathcal{R} \cup \{N g_j^{\overline{k}} \leftarrow N d_1 g_j, \ldots, N d_{k-1} g_j, N d_{k+1} g_j, \ldots, N d_N g_j\}$ *for* $k = 1, .., n$ *and* $j = 1, .., m$*;*
- $\mathcal{A}_D = \mathcal{A}$*;*
- $\mathcal{C}_D$ *is* $\mathcal{C}$ *with* $\mathcal{C}(N d_k g_j) = \{d_k g_j\}$ *replaced by* $\mathcal{C}(N d_k g_j) = \{d_k g_j, N g_j^{\overline{k}}\}$*, for* $k = 1, .., n$ *and* $j = 1, .., m$*.*

The intuition behind Definition 8 is as follows: a decision d_k is selected either if it meets all goals, or for goals that d_k does not meet, there is no other d' meeting them. Hence the contrary of $N d_k g_j$ (reads d_k does not meet g_j) is either $d_k g_j$ (d_k meets g_j) or $N g_j^{\overline{k}}$ (all decisions other than d_k do not meet g_j). The following theorem holds.

Theorem 3. *Given* $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, let* df_D *be the dominant ABA framework corresponding to* df*, then for all decisions* $d \in \mathtt{D}$*,* $d \in \psi_d(df)$ *iff* $\{d\} \vdash d$ *is admissible in* df_D*.*

Proof. (Sketch.) We first prove dominance implies admissibility for $d_k \in \mathtt{D}$. Since d_k is dominant, d_k meets all goals that is met by a decision in $\mathtt{D}$. Hence, for each goal g_j, either (1) there is $a_i \in \mathtt{A}$, such that $\mathtt{T_{DA}}[\mathtt{k}, \mathtt{i}] = \mathtt{T_{GA}}[\mathtt{j}, \mathtt{i}] = 1$ and $d_k a_i \leftarrow$ and $g_j a_i \leftarrow$ are in $\mathcal{R}$, therefore $\{\} \vdash d_k g_j$ exists and is not attacked; or (2) there is no argument $\{\} \vdash d_r g_j$ for all $d_r \in \mathtt{D}$ (g_j is not met by any d_r); therefore $\{N d_1 g_j, \ldots, N d_{k-1} g_j, N d_{k+1} g_j, N d_N g_j\} \vdash N g_j^{\overline{k}}$ is admissible. Whichever the case, $\{d_k\} \vdash d_k$ withstands attacks from $\{N d_k g_j\} \vdash N d_k g_j$, i.e. $N d_k g_j$ is always attacked. Moreover, since $\{N d_1 g_j, \ldots, N d_{k-1} g_j, N d_{k+1} g_j, N d_N g_j\} \cup \{d_k\}$ is also conflict-free, $\{d_k\} \vdash d_k$ is admissible.

We then show that admissibility implies dominance. Since $\{d_k\} \vdash d_k$ is admissible, all of its attackers must be counter attacked, i.e., $\{N d_k g_j\} \vdash N d_k g_j$ are attacked for all j. Each $N d_k g_j$ is attacked either because there is $\{d_k g_j\} \vdash d_k g_j$, or there is $\{N d_1 g_j, \ldots, N d_{k-1} g_j, N d_{k+1} g_j, N d_N g_j\} \vdash N g_j^{\overline{k}}$, i.e., either g_j is met by d_k or there is no $d' \in \mathtt{D}$ meeting g_j. Therefore d_k is dominant.

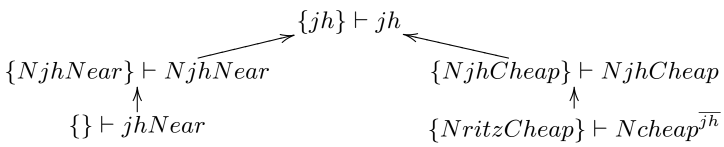

Fig. 2. Graphical illustration of ABA computation for dominant decisions.

We illustrate the ABA computation of dominant decisions in Figure 2. The dominant ABA framework corresponding to the decision framework shown in Example 2 is omitted due to the lack of space. It can be seen that $\{jh\} \vdash jh$ is admissible because (1) jh is *near*, hence $\{\} \vdash jhNear$ exists and not attacked; and (2) though jh is not *cheap*, hence there is no $\{\} \vdash jhCheap$ to attack $\{NjhCheap\} \vdash NjhCheap$, but and *ritz* is not *cheap* either, so $\{NritzCheap\} \vdash Ncheap^{\overline{jh}}$ exists and attacks $\{NjhCheap\} \vdash NjhCheap$.

Similarly, we can define *weakly dominant ABA framework* to compute weakly dominant decisions, as follows.

Definition 9. *Given* $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$, $|\mathtt{D}| = n$ *and* $|\mathtt{A}| = m$, *the* weakly dominant ABA framework corresponding to *df is* $df_W = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$, *where*

– $\mathcal{R}$ *is such that: for all* $k = 1, .., n; j = 1, .., m$ *and* $i = 1, .., l$:
  - *if* $\mathtt{T_{DA}}[\mathtt{k}, \mathtt{i}] = 1$ *then* $d_k a_i \leftarrow$;
  - *if* $\mathtt{T_{GA}}[\mathtt{j}, \mathtt{i}] = 1$ *then* $g_j a_i \leftarrow$;
  - $d_k g_j \leftarrow d_k a_i, g_j a_i$;

  for all $r, k = 1, .., n, r \neq k$; *and* $j = 1, .., m$:
  - $Sd_r d_k \leftarrow d_r g_j, Nd_k g_j, NSd_k d_r$;
  - $\overline{S}d_k d_r \leftarrow d_k g_j, Nd_r g_j$;

– $\mathcal{A}$ *is such that:* d_k, *for* $k = 1, .., n$;
  $NSd_k d_r$, *for* $r, k = 1, .., n, r \neq k$;
  $Nd_k g_j$, *for* $k = 1, .., n$ *and* $j = 1, .., m$;
– $\mathcal{C}$ *is such that:* $\mathcal{C}(d_k) = \{Sd_1 d_k, \ldots, Sd_{k-1} d_k, Sd_{k+1} d_k, \ldots, Sd_n d_k\}$, *for* $k = 1, .., n$;
  $\mathcal{C}(NSd_k d_r) = \{\overline{S}d_k d_r\}$, *for* $r, k = 1, .., n, r \neq k$;
  $\mathcal{C}(Nd_k g_j) = \{d_k g_j\}$, *for* $k = 1, .., n$ *and* $j = 1, .., m$.

The intuition behind Definition 9 is as follows: given a decision d_k in a decision framework, d_k is selected w.r.t. ψ_w iff there is no $d' \in \mathtt{D} \setminus \psi_w(df)$ such that the goals d' meets is a super-set of goals met by d_k. We test this for all $d' \neq d_k$ by letting the contrary of d_k be $\{Sd_1 d_k, \ldots, Sd_{k-1} d_k, Sd_{k+1} d_k, \ldots, Sd_n d_k\}$, standing for $\gamma(d_1) \supset \gamma(d_k), \ldots, \gamma(d_n) \supset \gamma(d_k)$. To "prove" $Sd_r d_k$, one needs to show two conditions: (1) there exists $g_j \in \mathtt{G}$, such that d_r meets g_j and d_k does not (hence "prove" $d_r g_j$ and $Nd_k g_j$); and (2) there does not exist $g'_j \in \mathtt{G}$, such that d_k meets g'_j and d_r does not (hence "prove" $NSd_k d_r$). Condition (1) is represented by having the first two terms in the body of the rule $Sd_r d_k \leftarrow d_r g_j, Nd_k g_j, NSd_k d_r$; condition (2) is represented by the last term in the body

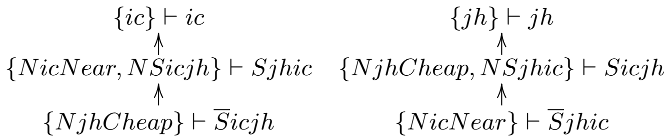

Fig. 3. Graphical illustration of ABA computation for weakly dominant decisions. The figure on the left can be read as follows. (1) Claiming *ic* is weakly dominant (the root argument). (2) *jh* is better, as *ic* is not near but *jh* is (the middle argument). (3) *jh* is not always better than *ic* as *jh* is not cheap but *ic* is (the bottom argument).

of this rule. To check NSd_kd_r, we need to fail at proving $\overline{S}d_kd_r$, which can only be proved by using the rule: $\overline{S}d_kd_r \leftarrow d_kg_j, Nd_rg_j$.

Similar to Theorem 2 and 3, the following theorem holds.

Theorem 4. *Given* $df = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$*, let* df_W *be the weakly dominant ABA framework corresponds to* df*. Then for all decisions* $d \in \mathtt{D}$*,* $d \in \psi_w(df)$ *iff* $\{d\} \vdash d$ *is admissible in* df_W*.*

Proof. (Sketch.) We first prove that weakly dominance implies admissibility for $d_k \in \mathtt{D}$. Since d_k is weakly dominant, then there is no $d_r \in \mathtt{D} \setminus \psi_w(df)$ such that $\gamma(d_k) \subset \gamma(d_r)$. Hence, given any $d_r \in \mathtt{D} \setminus \{d_k\}$, for each $g \in \gamma(d_r)$, either (1) $g \in \gamma(d_k)$ or (2) $g \notin \gamma(d_k)$, but there exists some $g' \in \mathtt{G}$ such that $g' \in \gamma(d_k)$ and $g' \notin \gamma(d_r)$. If it is case (1), then Nd_kg_j does not hold as d_k meets g_j; if it is case (2), then NSd_kd_r does not hold as there is some g' met by d_k but not d_r. Hence, whichever the case, arguments for Sd_rd_k are either nonexistent (case 1) or are counterattacked (case 2). Since the contrary of d_k is $\{Sd_rd_k\}$ for all $r \neq k$, and to build an argument for Sd_rd_k one needs to show both Nd_kg_j and NSd_kd_r, failing at constructing arguments for Sd_rd_k and $\{d_k, Nd_rg_t\}$ being conflict-free jointly make $\{d_k\} \vdash d_k$ admissible.

Then we show that if $\{d_k\} \vdash d_k$ is admissible, then d_k is weakly dominant. Since $\{d_k\} \vdash d_k$ is admissible, all of its attackers are counterattacked or nonexistent. Hence, arguments for Sd_kd_r are either counterattacked or nonexistent for all $d_r \neq d_k$. Since $Sd_rd_k \leftarrow d_rg_j, Nd_kg_j, NSd_kd_r$, if an argument for Sd_kd_r does not exist, it means there is no d_rg_j, hence d_r does not meet g_j. If an argument for Sd_kd_r exists but counterattacked, it means either (1) Nd_kg_j is attacked by d_kg_j or (2) NSd_kd_r is attacked by $\overline{S}d_kd_r$. In case (1), either both d_k and d_r meet g_j or d_r does not meet it. In case (2), there is some g' such that g' is met by d_k but not d_r. Whichever the case, $\gamma(d_k)$ is not a subset of $\gamma(d_r)$. Therefore d_k is weakly dominant.

We illustrate the ABA computation of weakly dominant decisions in Figure 3. The weakly dominant ABA framework corresponding to the decision framework is omitted. It can be seen that both $\{ic\} \vdash ic$ and $\{jh\} \vdash jh$ are admissible because jh is *near* but not *cheap* and ic is *cheap* but not *near*. Hence each of the two meets some goal that is not met by the other. $ritz$ is not weakly dominant, as it is neither *cheap* nor *near*.

5 Decisions with Preferences

Thus far, we present a decision framework characterised by two tables, $\mathtt{T_{DA}}$ and $\mathtt{T_{GA}}$, describing the relations between decisions, attributes and goals. However, in cases where not all goals are considered equal, and there are multiple decisions meeting different goals (i.e., a decision framework with only weakly dominant decisions but no dominant or strongly dominant decision) it is useful to consider *preferences* over goals upon selecting decisions. We extend our decision framework to include preferences and define *extended decision frameworks* as follows.

Definition 10. *An* extended decision framework *is a tuple* $\langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}}, \mathtt{P} \rangle$*, in which* $\langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}} \rangle$ *forms a decision framework and* $\mathtt{P}$ *is a partial order over goals, representing the preference ranking of goals.*

We use $\mathcal{EDF}$ *to denote the set of possible extended decision frameworks.*

We represent $\mathtt{P}$ as a set of constraints $g_i > g_j$ for $g_i, g_j \in \mathtt{G}$. Extended decision frameworks are generalisation of decision frameworks as any decision framework can be considered as an extended decision framework with a uniformly equal preference order.

Example 6 illustrate the notion of extended decision framework as follows.

Example 6. We reuse Example 3 but remove *ritz* in this example. We let $\mathtt{T_{DA}}$ be the first two rows and $\mathtt{T_{GA}}$ remain the same. We also add the preference ranking: $\{near > cheap\}$.

We do not redefine Definition 2 for extended decision frameworks as this definition remains the same in extended decision frameworks.

We need to redefine *extended decision functions* over extended decision frameworks to select decisions. Formally,

Definition 11. *An* extended decision function *is a mapping* $\psi^E : \mathcal{EDF} \mapsto 2^{\mathcal{DEC}}$*, such that, given* $edf = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}}, \mathtt{P} \rangle$*,* $\psi^E(edf) \subseteq \mathtt{D}$*. For any* $d, d' \in \mathtt{D}$*, if* $\gamma(d) = \gamma(d')$ *and* $d \in \psi^E(df)$*, then* $d' \in \psi^E(df)$*. We say that* $\psi^E(edf)$ *are* selected *w.r.t.* ψ^E*.*

We use Ψ^E *to denote the set of all extended decision functions.*

More specifically, we define *best possible extended decision function* to select the decision that meets the most preferred goal that is ever met by any decision.

Definition 12. *A* best possible decision function $\psi^E \in \Psi^E$ *is that given* $edf = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}}, \mathtt{P} \rangle$*, for all* $d \in \mathtt{D}$*, if* $d \in \psi^E(edf)$*, then (1) there is some* $g \in \gamma(d)$*, and (2) there is no* $g' \in \gamma(d')$ *for all* $d' \in \mathtt{D} \setminus \{d\}$*, such that* $g' > g$ *in* $\mathtt{P}$*. We say* d *is a* best possible decision*.*

We refer to a generic best possible decision function as ψ^E_b*.*

Given the extended decision framework *edf* shown in Example 6, since *jh* meets the top preference goal, *near*, *jh* is a best possible decision in *edf*. Neither *ic* nor *ritz* meets *near*, so neither of the two is a best possible decision.

We can use ABA to compute best possible decisions in an extended decision framework as well, as follows:

Definition 13. *Given edf* $= \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}}, \mathtt{P} \rangle$*, the* best possible ABA framework corresponding to *edf is* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$*, where*

- $\mathcal{R}$ *is such that:*
  for all $k = 1, .., n; j = 1, .., m$ *and* $i = 1, .., l$*:*
  - *if* $\mathtt{T_{DA}}[\mathtt{k}, \mathtt{i}] = 1$ *then* $d_k a_i \leftarrow$*;*
  - *if* $\mathtt{T_{GA}}[\mathtt{j}, \mathtt{i}] = 1$ *then* $g_j a_i \leftarrow$*;*
  - $d_k g_j \leftarrow d_k a_i, g_j a_i$*;*

  for all g_1, g_2 *in* $\mathtt{G}$*, if* $g_1 > g_2 \in \mathtt{P}$*, then* $Pg_1g_2 \leftarrow$*;*
  for all $k = 1, .., n; j = 1, .., m$*:* $d_k \leftarrow d_k g_j, Nsmp_j^k$*;*
  for all $k, r = 1, .., n, k \neq r; j, t = 1, .., m, j \neq t$*:* $smp_j^k \leftarrow d_r g_t, Pg_t g_j$*.*
- $\mathcal{A}$ *is such that:* $Nsmp_j^k$*, for all* $k = 1, .., n; j = 1, .., m$*;*
- $\mathcal{C}$ *is such that:* $\mathcal{C}(Nsmp_j^k) = \{smp_j^k\}$*, for all* $k = 1, .., n; j = 1, .., m$*.*

The intuition behind Definition 13 is as follows: in order to let d_k be the best possible decision, d_k needs to meet $g_j \in \mathtt{G}$, such that there is no other $d' \in \mathtt{D}$ meeting $g' \in \mathtt{G}$ and $g' > g$ in $\mathtt{P}$. Hence we have the rule $d_k \leftarrow d_k g_j, Nsmp_j^k$, standing for d_k meets g_j ($d_k g_j$) and "there is no other decision meeting goals more preferred than d_k meeting g_j" ($Nsmp_j^k$).

We illustrate best possible ABA framework in the following example.

Example 7. The best possible ABA framework corresponds to the extended decision framework shown in Example 6 is follows.
$\mathcal{R}$:

$$
\begin{array}{ll}
jhCheap \leftarrow jh50, cheap50 & jhNear \leftarrow jh50, near50 \\
jhCheap \leftarrow jh70, cheap70 & jhNear \leftarrow jh70, near70 \\
jhCheap \leftarrow jh200, cheap200 & jhNear \leftarrow jh200, near200 \\
jhCheap \leftarrow jhSK, cheapSK & jhNear \leftarrow jhSK, nearSK \\
jhCheap \leftarrow jhPic, cheapPic & jhNear \leftarrow jhPic, nearPic \\
icCheap \leftarrow ic50, cheap50 & icNear \leftarrow ic50, near50 \\
icCheap \leftarrow ic70, cheap70 & icNear \leftarrow ic70, near70 \\
icCheap \leftarrow ic200, cheap200 & icNear \leftarrow ic200, near200 \\
icCheap \leftarrow icSK, cheapSK & icNear \leftarrow icSK, nearSK \\
icCheap \leftarrow icPic, cheapPic & icNear \leftarrow icPic, nearPic \\
jh \leftarrow jhCheap, Nsmp_{cheap}^{jh} & jh \leftarrow jhNear, Nsmp_{near}^{jh} \\
ic \leftarrow icCheap, Nsmp_{cheap}^{ic} & ic \leftarrow icNear, Nsmp_{near}^{ic}
\end{array}
$$

$$
\begin{array}{ll}
smp_{cheap}^{jh} \leftarrow icNear, PNearCheap & smp_{near}^{jh} \leftarrow icCheap, PCheapNear \\
smp_{cheap}^{ic} \leftarrow jhNear, PNearCheap & smp_{near}^{ic} \leftarrow jhCheap, PCheapNear
\end{array}
$$

$$
PNearCheap \leftarrow \quad jh70 \leftarrow \quad jhSK \leftarrow \quad ic50 \leftarrow \quad cheap50 \leftarrow \quad nearSK \leftarrow
$$

$\mathcal{A}$:

$$
Nsmp_{cheap}^{jh} \qquad Nsmp_{near}^{jh} \qquad Nsmp_{cheap}^{ic} \qquad Nsmp_{near}^{ic}
$$

$\mathcal{C}$:

$$
\begin{array}{ll}
\mathcal{C}(Nsmp_{cheap}^{jh}) = \{smp_{cheap}^{jh}\} & \mathcal{C}(Nsmp_{near}^{jh}) = \{smp_{near}^{jh}\} \\
\mathcal{C}(Nsmp_{cheap}^{ic}) = \{smp_{cheap}^{ic}\} & \mathcal{C}(Nsmp_{near}^{ic}) = \{smp_{near}^{ic}\}
\end{array}
$$

Theorem 5. *Given an extended decision framework* $edf = \langle \mathtt{D}, \mathtt{A}, \mathtt{G}, \mathtt{T_{DA}}, \mathtt{T_{GA}}, \mathtt{P} \rangle$, *let* edf_b *be the best possible ABA framework corresponding to* edf. *Then, for all* $d \in \mathtt{D}$, $d \in \psi_b^E(edf)$ *iff* d *is the claim of an admissible argument in* edf_b.

Proof. (Sketch.) We first show that if $d_k \in \mathtt{D}$ is a best possible decision, then there is an admissible argument $\{Nsmp_j^k\} \vdash d_k$. Since d_k is a best possible decision, it meets $g_j \in \mathtt{G}$ (we hence have $d_k g_j$), and there is no other $g_t \in \mathtt{G}$, such that $g_t > g_j \in \mathtt{P}$ and $g_t \in \gamma(d_r)$ for some $d_r \in \mathtt{D} \setminus \{d_k\}$. Hence, there is no argument for smp_j^k and since $\{Nsmp_j^k\}$ is conflict-free, $\{Nsmp_j^k\} \vdash d_k$ is not attacked hence is admissible.

To show that d_k is a best possible decision given $\{Nsmp_j^k\} \vdash d_k$ being admissible, we need to show there is no $d' \in \mathtt{D} \setminus \{d_k\}$ such that d' meets a more preferred goal than d_k meeting $g_j \in \mathtt{G}$. Since $\{Nsmp_j^k\} \vdash d_k$ is admissible, it withstands its attacks. Since $\mathcal{C}(Nsmp_j^k) = \{smp_j^k\}$ and arguments for smp_j^k are not supported by any assumptions (no assumptions in rules: $smp_j^k \leftarrow d_r g_t, P g_t g_j$; $d_k g_j \leftarrow d_k a_i, g_j a_i$; $d_k a_i \leftarrow$; $g_j a_i \leftarrow$ and $P g_1 g_2 \leftarrow$), $\{Nsmp_j^k\} \vdash d_k$ withstanding its attacks means there is no argument for smp_j^k. Therefore it is not the case that there exists $g' \in \mathtt{G}$ and $d' \in \mathtt{D} \setminus \{d\}$, such that $g' > g_j$ and $g' \in \gamma(d')$. Hence d_k is a best possible decision.

In Example 7, we "prove" jh using the rule $jh \leftarrow jhNear, Nsmp_{near}^{jh}$. We "prove" $jhNear$ with rules $jhNear \leftarrow jhSK, nearSK$; $jhSK \leftarrow$ and $nearSK \leftarrow$. Since $Nsmp_{near}^{jh}$ is an assumption, we need to show it withstands all attacks. The contrary of $Nsmp_{near}^{jh}$ is $\{smp_{near}^{jh}\}$, which can only be "proved" using the rule $smp_{near}^{jh} \leftarrow icCheap, PCheapNear$. However, since there is no rule for $PCheapNear$, there is no argument for smp_{near}^{jh}. Therefore $Nsmp_{near}^{jh}$ is not attacked and $\{Nsmp_{near}^{jh}\} \vdash jh$ is admissible.

6 Related Work

Amgoud and Prade [1] present a formal model for making decisions using abstract argumentation. Our work differs from theirs as: (1) they use abstract argumentation whereas we use ABA; (2) they use a pair-wise comparison between decisions to select the "winning" decision whereas we use an unified process to map decision frameworks into ABA and then compute admissible arguments.

Matt et.al. [8] present an ABA based decision making model. Our work differs from theirs as: (1) we have studied three different notions of dominant decisions whereas they have studied one; (2) we have studied decision making with preference whereas they have not.

Black and Atkinson [2] present a multi-agent dialogue model for agent to decide actions jointly. Our work differs from theirs as: (1) we have focused on ABA based decision making whereas they have studied a dialogue model; (2) we have studied several different decision criteria where they have not.

Dung et. al. [4] present an argumentation-based approach to contract negotiation. Part of that work can also be viewed as argumentation-based decision-making taking preferences into account. The main differences between that work and ours are: (1) we give formal definition of decision making frameworks whereas they do not; (3) we make explicit connections between "good" decisions and "acceptable" arguments whereas they do not.

7 Conclusion

We present a formal model for decision making with ABA. In this model, we represent agents' knowledge in decision frameworks, which capture relations between decisions, goals, and attributes, e.g., decisions meeting goals, goals being satisfied by attributes. We then define decision functions to model different decision criteria. We define decision functions that select decisions meeting all goals, most goals, goals no others met, and most preferred achievable goals. We then map both decision frameworks and decision functions into ABA frameworks. In this way, computing selected decisions becomes computing admissible arguments. We obtain sound and complete results such as selected decisions are claims of admissible arguments and vice versa. The main advantage of our approach is that it gives an argumentation-based justification of selected decisions, while finding them.

Future directions include (1) further studying of decision criteria / functions for decision making with preference; (2) studying decision making with other form of knowledge representation (not limited to tables), (3) linking to existed decision theoretic work, and (4) studying decision making in the context of multiple agents, in which agents sharing potentially conflicting knowledge and preferences.

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